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The Influence of Pad Flexibility on the Dynamic Coefficients of a Tilting Pad Journal Bearing

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An approximate method is developed to include the flexibility of the pad in the calculation of the stiffness and damping properties of a tilting pad journal bearing. It is a small-amplitude perturbation solution in which the pad deformation is accounted for solely by the change in clearance. A comparison of results with those obtained from a more complete elasto-hydrodynamic solution shows good agreement.

Introduction

Experience suggests that tilting pad journal bearings have less damping than predicted from theory. A major reason is that the theory assumes the pads to be rigid whereas a significant reduction in damping does occur when the pads are treated as flexible. Thus, for more realistic results the flexibility of the pad and, also, of the pivot should be included in the analysis as clearly demonstrated by references [1, 2, 14].

The referenced studies employ an elasto-hydrodynamic solution with coupling between the Reynolds equation and the elasticity equation via the local pressure and the local change in film thickness ([2] unjustifiably ignores the dynamically induced deformations). This necessitates an iterative procedure which is frequently slow in converging and, therefore, time-consuming.

Instead, an alternative method has been developed. It is approximate, but much faster, which is of practical importance when performing rotor-dynamics calculations.

Analysis

To consider a single pad, its radius of curvature is $R+C$ where R is the journal radius, and C is the machined clearance plus any contribution from thermal deformations. The static load on the pad causes the clearance to increase by an amount ΔC_0 such that the operating clearance becomes:

$$C_0 = C + \Delta C_0 \tag{1}$$

For later use the ratio between the two clearances is given as

$$\Gamma = \frac{C}{C_0} \tag{2}$$

Under dynamic conditions there is a further time varying change in clearance, ΔC.

The journal center position relative to the pad is measured in an x-y coordinate system with origin in the pad's center of curvature and with the x-axis passing through the pivot point, see Fig. 1. The static equilibrium position has the coordinates x_0 and y_0, and the dynamic amplitudes Δx and Δy are measured from this point. With θ as the angular coordinate, measured from the negative x-axis such that the pivot point is at $\theta = \pi$, the oil film thickness, h, can be expressed in dimensionless form as:

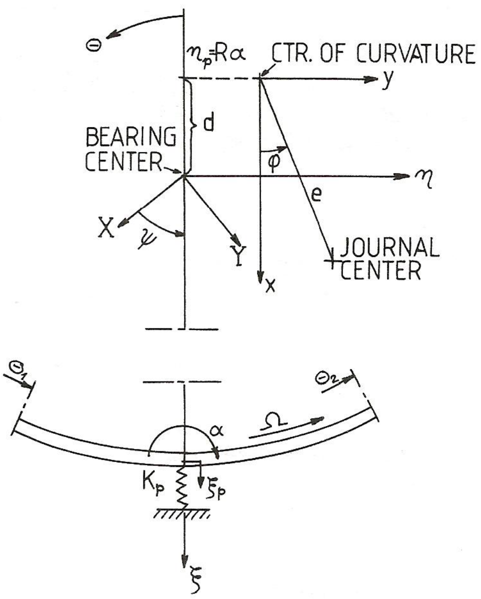

Fig. 1 Coordinate systems

$$\bar{h} = \frac{h}{C_0} = \bar{h}_0 + \Delta\bar{h} \tag{3}$$

where:

$$\bar{h}_0 = 1 + \Gamma(\bar{x}_0\cos\theta + \bar{y}_0\sin\theta) = 1 + \epsilon_0\cos(\theta - \phi) \tag{4}$$

$$\Delta\bar{h} = \Gamma/\Delta\bar{C} + \Delta\bar{x}\cos\theta + \Delta\bar{y}\sin\theta) \tag{5}$$

and

$$\bar{x}_0, \bar{y}_0 = x_0/C, y_0/C$$
$$\Delta\bar{C}, \Delta\bar{x}, \Delta\bar{y} = \Delta C/C, \Delta x/C, \Delta y/C \tag{6}$$

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The pressure, p, in the oil film is determined from Reynolds equation which in dimensionless form is written as:

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\frac{\partial}{\partial\zeta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\zeta}\right)=\frac{\partial\bar{h}}{\partial\theta}+2\frac{\partial\bar{h}}{\partial\tau} \tag{7}$$

where:

$$\bar{p}=p/6\mu\Omega\left(\frac{R}{C_0}\right)^2 \tag{8}$$

$$\zeta=z/R$$

$$\tau=\Omega t$$

μ is the oil viscosity, Ω is the angular speed of rotation, z is the axial coordinate, and t is time.

A perturbation solution of equation (7) is sought where:

$$\bar{p}=\bar{p}_0+\Delta\bar{p} \tag{9}$$

$$\Delta\bar{p}=\Gamma\,(\bar{p}_x\Delta\bar{x}+\bar{p}_x'\Delta\dot{\bar{x}}+\bar{p}_y\Delta\bar{y}+\bar{p}_y'\Delta\dot{\bar{y}}+\bar{p}_c\Delta\bar{C}+\bar{p}_c'\Delta\dot{\bar{C}}) \tag{10}$$

Dots indicate derivative with respect to τ.

By substituting equations (3), (5), (9) and (10), into equation (7) and retaining only first-order terms, seven equations are obtained:

$$R\{\bar{p}_0\}=\frac{\partial\bar{h}_0}{\partial\theta}=\Gamma(-\bar{x}_0\sin\theta+\bar{y}_0\cos\theta)=-\epsilon_0\sin(\theta-\phi) \tag{11}$$

$$R\{\bar{p}_x\}=-3\,\frac{\cos\theta}{\bar{h}_0}\,\frac{\partial\bar{h}_0}{\partial\theta}-3\bar{h}_0^3\,\frac{\partial\bar{p}_0}{\partial\theta}\,\frac{\partial}{\partial\theta}\left(\frac{\cos\theta}{\bar{h}_0}\right)-\sin\theta \tag{12}$$

$$R\{\bar{p}_x'\}=2\cos\theta \tag{13}$$

$$R\{\bar{p}_y\}=-3\,\frac{\sin\theta}{\bar{h}_0}\,\frac{\partial\bar{h}_0}{\partial\theta}-3\bar{h}_0^3\,\frac{\partial\bar{p}_0}{\partial\theta}\,\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{\bar{h}_0}\right)+\cos\theta \tag{14}$$

$$R\{\bar{p}_y'\}=2\sin\theta \tag{15}$$

$$R\{\bar{p}_c\}=-\frac{3}{\bar{h}_0}\,\frac{\partial\bar{h}_0}{\partial\theta}-3\bar{h}_0^3\,\frac{\partial\bar{p}_0}{\partial\theta}\,\frac{\partial}{\partial\theta}\left(\frac{1}{\bar{h}_0}\right) \tag{16}$$

$$R\{\bar{p}_c'\}=2 \tag{17}$$

where the left-hand side operator is:

$$R\{\ \}=\frac{\partial}{\partial\theta}\left(\bar{h}_0^3\,\frac{\partial}{\partial\theta}\right)+\frac{\partial}{\partial\zeta}\left(\bar{h}_0^3\,\frac{\partial}{\partial\zeta}\right) \tag{18}$$

The equations are solved numerically in finite difference form as for example shown in details in reference [12].

The reaction forces, F_x and F_y, are obtained by integrating the pressure distribution over the pad arc, from $\theta=\theta_1$ to $\theta=\theta_2$, and over the axial length L:

$$\bar{F}_x=F_x/\mu NDL\left(\frac{R}{C}\right)^2=\bar{F}_{x0}+\Delta\bar{F}_x$$

$$\bar{F}_y=F_y/\mu NDL\left(\frac{R}{C}\right)^2=\bar{F}_{y0}+\Delta\bar{F}_y \tag{19}$$

where:

$$\left.\begin{matrix}\bar{F}_{x0}\\ \bar{F}_{y0}\end{matrix}\right\}=\Gamma^2\left(-\frac{6\pi}{L/D}\int_{\theta_1}^{\theta_2}\int_0^{L/D}\bar{p}_0\left\{\begin{matrix}\cos\theta\\ \sin\theta\end{matrix}\right\}d\zeta d\theta\right) \tag{20}$$

D is the diameter ($D=2R$) and N is the speed in rps ($N=\Omega/2\pi$). Usually $\bar{p}_0$ is calculated for some selected value of $\Gamma\bar{x}_0=x_0/C_0$ while $\Gamma\bar{y}_0$ is varied until $\bar{F}_{y0}=0$ which, then, is the static equilibrium position. The corresponding value of $\bar{F}_{x0}$ represents the inverse Sommerfeld number.

The dynamic forces may be written as:

$$\Delta\bar{F}_x=\Gamma^3(\bar{K}_{xx}'\Delta\bar{x}+\bar{B}_{xx}'\Delta\dot{\bar{x}}+\bar{K}_{xy}'\Delta\bar{y}+\bar{B}_{xy}'\Delta\dot{\bar{y}}+\bar{K}_{xc}'\Delta\bar{C}+\bar{B}_{xc}'\Delta\dot{\bar{C}})$$

$$\Delta\bar{F}_y=\Gamma^3(\bar{K}_{yx}'\Delta\bar{x}+\bar{B}_{yx}'\Delta\dot{\bar{x}}+\bar{K}_{yy}'\Delta\bar{y}+\bar{B}_{yy}'\Delta\dot{\bar{y}}+\bar{K}_{yc}'\Delta\bar{C}+\bar{B}_{yc}'\Delta\dot{\bar{C}}) \tag{21}$$

where:

$$\left.\begin{matrix}\bar{K}_{xx}'\\ \bar{K}_{yx}'\end{matrix}\right\}=-\frac{6\pi}{L/D}\int_{\theta_1}^{\theta_2}\int_0^{L/D}\bar{p}_x\left\{\begin{matrix}\cos\theta\\ \sin\theta\end{matrix}\right\}d\zeta d\theta \tag{22}$$

and analogously for the remaining coefficients.

The change in clearance, $\Delta C_0+\Delta C$, caused by the pressure distribution, is evaluated by an approximate method. The pad is treated as a beam with a load per unit length of:

$$q(\theta)=2\int_0^{L/D}pdz \tag{23}$$

This sets up a bending moment, M, in the pad:

Nomenclature

- C = radial clearance of unloaded pad, including thermal deformations
- C_0 = $C+\Delta C_0$, operating radial clearance of pad
- ΔC_0 = change in radial clearance caused by static load
- ΔC = change in radial clearance caused by dynamic load
- D = journal diameter
- d_0 = preload of unloaded pad, including thermal deformations
- E = modulus of elasticity
- e = journal center eccentricity from center of curvature of pad
- h_{piv} = oil film thickness at pivot point
- I = cross-sectional area moment of inertia of pad
- I_0 = reference value of I
- J = mass moment of inertia of pad in pitching
- K_p = dynamic radial stiffness of pivot
- L = axial length of pad
- m = mass of pad
- N = rotational speed, rps
- p = oil film pressure
- R = journal radius
- R_m = radius of curvature of neutral axis of pad
- t = time
- W = static load on bearing
- X,Y = coordinate system, origin in bearing center, X-axis in load direction
- x,y = coordinate system for pad, origin in center of curvature
- α = $\alpha_0+\Delta\alpha$, tilt angle of pad
- Γ = C/C_0
- γ = ω/Ω, frequency ratio
- ϵ_0 = e_0/C_0, static eccentricity ratio
- ξ,η = coordinate system for pad, origin in bearing center
- ξ_p = $\xi_{p0}+\Delta\xi_p$, radial motion of pad
- η_p = $R\alpha=\eta_{p0}+\Delta\eta_p$, tangential motion of center of curvature of pad
- $\theta_2-\theta_1$ = pad arc
- Λ = dimensionless flexibility parameter, equation (28)
- μ = oil viscosity
- τ = Ωt, dimensionless time
- ϕ = attitude angle
- ψ = angle between X-axis and ξ-axis
- Ω = $2\pi N$, angular speed of rotation
- ω = angular whirl frequency

$$M = \begin{cases} \int_{\theta_1}^{\theta} q(\theta')R_m \sin(\theta-\theta')Rd\theta' & \underline{\theta<\pi} \\ \\ \int_{\theta}^{\theta_2} q(\theta')R_m \sin(\theta'-\theta)Rd\theta' & \underline{\theta>\pi} \end{cases} \quad (24)$$

where R_m is the radius of curvature of the neutral axis of the pad. With a cross-sectional area moment of inertia, I, and an elasticity modulus, E, the averaged change in curvature is given by:

$$\frac{1}{R_m} - \frac{1}{R_m + \Delta C_0 + \Delta C} \simeq \frac{\Delta C_0 + \Delta C}{R_m^2} = \frac{1}{\theta_2 - \theta_1}\int_{\theta_1}^{\theta_2} \frac{M}{EI} d\theta \quad (25)$$

With substitution from equations (9), (10), (23), and (24), the result may be written as:

$$\frac{\Delta C_0}{C} = \Gamma^2 \Lambda a_0 \quad (26)$$

$$\frac{\Delta C}{C} = \Delta \bar{C} = \Gamma^3 \Lambda (a_x \Delta\dot{\bar{x}} + b_x \Delta \bar{x} + a_y \Delta \bar{y} + b_y \Delta \dot{\bar{y}} + a_c \Delta \bar{C} + b_c \Delta \dot{\bar{C}}) \quad (27)$$

where:

$$\Lambda = \mu NDL \left(\frac{R}{C}\right)^2 \cdot \frac{R_m^2}{EI_0} \cdot \frac{R_m}{C} \quad (28)$$

I_0 is some representative value of I.

The coefficients are computed from:

$$a_0 = \frac{1}{\theta_2 - \theta_1} \cdot \frac{6\pi}{L/D} \cdot \left[\int_{\theta_1}^{\pi} \frac{I_0}{I} \int_{\theta_1}^{\theta} \sin(\theta - \theta') \int_0^{L/D} \bar{p}_0 d\zeta d\theta' d\theta + \int_{\pi}^{\theta_2} \frac{I_0}{I} \int_{\theta}^{\theta_2} \sin(\theta' - \theta) \int_0^{L/D} \bar{p}_0 d\zeta d\theta' d\theta \right] \quad (29)$$

and analogously for the remaining coefficients (as an example a_x is obtained from equation (29) by replacing $\bar{p}_0$ with $\bar{p}_x$).

The foregoing derivation assumes the pad to be in equilibrium. This requires that at the pivot point there is a vertical reaction force, F_x, a horizontal reaction force, F_y, and a reaction moment. The two forces are given by equations (19), (20), and (21), and the reaction moment is found to be $R_m F_y$, equal to the discontinuity in the bending moment at $\theta = \pi$ as deduced from equation (24). Under static conditions, F_y is zero and there is no discontinuity, but this may not be true under dynamic conditions. The dynamic moment, $R_m \cdot \Delta F_y$, however, is resisted only by the inertia of the pad, and setting it equal to zero implies that the contribution from the inertia forces to the deformation of the pad has been ignored.

By making use of equations (1) and (2), equation (26) is written as:

$$\frac{\Delta C_0}{C}\left(1 + \frac{\Delta C_0}{C}\right)^2 = \Lambda a_0 \quad (30)$$

which can be solved for $\Delta C_0 / C$, whereby the value of Γ is established.

For the dynamic case the motion shall be assumed to be harmonic with an angular frequency ω such that:

$$\Delta x = \Delta x_c \cos\omega t - \Delta x_s \sin\omega t = \mathrm{Re}\{(\Delta x_c + \Delta x_s)e^{i\omega t}\} \quad (31)$$

and similarly for Δy and ΔC ($i = \sqrt{-1}$ is the imaginary unit). This is written in short form as:

$$\Delta x = \Delta x_c + i \Delta x_s \quad (32)$$

where $\exp(i\omega t)$ and the real part operator are implied. Hence:

$$\frac{d\Delta x}{dt} = i\omega \Delta x \quad (33)$$

or in dimensionless form:

$$\Delta\dot{\bar{x}} = \frac{d\Delta\bar{x}}{dt} = i\gamma \Delta \bar{x} \quad (34)$$

where:

$$\gamma = \frac{\omega}{\Omega} \quad (35)$$

Thereby the solution of equation (27) becomes:

$$\Delta \bar{C} = Z'_{cx} \Delta \bar{x} + Z'_{cy} \Delta \bar{y} \quad (36)$$

where:

$$\begin{aligned} Z'_{cx} &= \Gamma^3 \Lambda (a_x + i\gamma b_x)/[1 - \Gamma^3 \Lambda (a_c + i\gamma b_c)] \\ Z'_{cy} &= \Gamma^3 \Lambda (a_y + i\gamma b_y)/[1 - \Gamma^3 \Lambda (a_c + i\gamma b_c)] \end{aligned} \quad (37)$$

Similarly the dynamic reaction forces from equation (21) can be expressed as:

$$\begin{aligned} \Delta \bar{F}_x &= \Gamma^3 (Z'_{xx} \Delta \bar{x} + Z'_{xy} \Delta \bar{y} + Z'_{xc} \Delta \bar{C}) \\ \Delta \bar{F}_y &= \Gamma^3 (Z'_{yx} \Delta \bar{x} + Z'_{yy} \Delta \bar{y} + Z'_{yc} \Delta \bar{C}) \end{aligned} \quad (38)$$

where:

$$Z'_{xx} = \bar{K}'_{xx} + i\gamma \bar{B}'_{xx} \quad (39)$$

and analogously for the remaining impedances.

Because the pad's center of curvature is not stationary, it becomes necessary to refer the journal center motion to a fixed ξ-η-coordinate system with origin in the bearing center and the ξ-axis passing through the pivot point. The distance between the two centers is d. The pad tilts the angle α around the pivot point and, in addition, it has a radial motion ξ_p because of the flexibility of the pivot. Referring to Fig. 1 it is seen that:

$$\begin{aligned} x &= \xi + d = e\cos\phi \\ y &= \eta - R\alpha = e\sin\phi \end{aligned} \quad (40)$$

For the unloaded pad, d equals d_0 which is the installed preload, including thermal deformations. With load on the pad, d increases by the increase in clearance, $\Delta C_0 + \Delta C$, and decreases by the radial motion ξ_p. Hence, for static conditions equation (40) may be written in dimensionless form as:

$$\begin{aligned} \bar{x}_0 &= \bar{\xi}_0 + \bar{d}_0 + \Delta \bar{C}_0 - \bar{\xi}_{p0} = \epsilon_0 \cos\phi_0 / \Gamma \\ \bar{y}_0 &= \bar{\eta}_0 - \bar{\eta}_{p0} = \epsilon_0 \sin\phi_0 / \Gamma \end{aligned} \quad (41)$$

where:

$$\bar{\xi}_0, \bar{\eta}_0, \bar{\xi}_{p0}, \Delta\bar{C}_0, \bar{d}_0 = \xi_0/C, \eta_0/C, \xi_{p0}/C, \Delta C_0/C, d_0/C \quad (42)$$

$$\bar{\eta}_{p0} = R\alpha_0 / C \quad (43)$$

For the dynamic load conditions, equations (40) yield the dimensionless equations:

$$\Delta \bar{x} = \Delta \bar{\xi} + \Delta \bar{C} - \Delta \bar{\xi}_p \quad (44)$$

$$\Delta \bar{y} = \Delta \bar{\eta} - \Delta \bar{\eta}_p \quad (45)$$

By substituting equation (36) into equation (44) it is found that:

$$\Delta \bar{x} = (\Delta \bar{\xi} - \Delta \bar{\xi}_p + Z'_{cy} \Delta \bar{y}) / (1 - Z'_{cx}) \quad (46)$$

Furthermore, equation (36) can be substituted into equations (38), and by making use of equations (46) and (45), the dynamic forces can be expressed as:

$$\begin{aligned} \Delta \bar{F}_x &= Z'_{\xi\xi} (\Delta \bar{\xi} - \Delta \bar{\xi}_p) + Z'_{\xi\eta} (\Delta \bar{\eta} - \Delta \bar{\eta}_p) \\ \Delta \bar{F}_y &= Z'_{\eta\xi} (\Delta \bar{\xi} - \Delta \bar{\xi}_p) + Z'_{\eta\eta} (\Delta \bar{\eta} - \Delta \bar{\eta}_p) \end{aligned} \quad (47)$$

where:

$$\begin{aligned} Z'_{\xi\xi} &= \Gamma^3 (Z'_{xx} + Z'_{xc} Z'_{cx}) / (1 - Z'_{cx}) \\ Z'_{\xi\eta} &= \Gamma^3 (Z'_{xy} + Z'_{xc} Z'_{cy}) + Z'_{cy} Z'_{\xi\xi} \end{aligned}$$

$$Z'_{\eta\xi}=\Gamma^3(Z'_{yx}+Z'_{yc}Z'_{cx})/(1-Z'_{cx})$$
$$Z'_{\eta\eta}=\Gamma^3(Z'_{yy}+Z'_{yc}Z'_{cy})+Z'_{cy}Z'_{\eta\xi} \tag{48}$$

The equations of motion for the pad are:

$$m\frac{d^2\Delta\xi_p}{dt^2}+K_p\Delta\xi_p=\Delta F_x$$
$$J\frac{d^2\Delta\alpha}{dt^2}=R\bullet\Delta F_y \tag{49}$$

where m is the mass of the pad, J is the pitch mass moment of inertia around the pivot axis, and K_pi the dynamic stiffness of the pivot. In dimensionless form the equations become:

$$(\bar{K}_p-\gamma^2\bar{m})\Delta\bar{\xi}_p=\Delta\bar{F}_x$$
$$-\gamma^2\bar{J}\bullet\Delta\bar{\eta}_p=\Delta\bar{F}_y \tag{50}$$

where:

$$\bar{K}_p=CK_p/\mu NDL\left(\frac{R}{C}\right)^2 \tag{51}$$

$$\bar{m}=C\,\Omega^2m/\mu NDL\left(\frac{R}{C}\right)^2 \tag{52}$$

$$\bar{J}=C\,\Omega^2\frac{J}{R^2}/\mu NDL\left(\frac{R}{C}\right)^2 \tag{53}$$

With substitution from equation (47), equation (50) may be solved:

$$\begin{Bmatrix}\Delta\bar{F}_x\\ \Delta\bar{F}_y\end{Bmatrix}=\begin{Bmatrix}Z'_{\xi\xi} & Z'_{\xi\eta}\\ Z'_{\eta\xi} & Z'_{\eta\eta}\end{Bmatrix}\frac{1}{\Delta}\begin{Bmatrix}(Z'_{\xi\xi}-\gamma^2\bar{J}) & -Z'_{\xi\eta}\\ -Z'_{\eta\xi} & (Z'_{\xi\xi}+\bar{K}_p-\gamma^2\bar{m})\end{Bmatrix}\begin{Bmatrix}\bar{K}_p-\gamma^2\bar{m}) & 0\\ 0 & -\gamma^2\bar{J}\end{Bmatrix}\begin{Bmatrix}\Delta\xi\\ \Delta\eta\end{Bmatrix} \tag{54}$$

where:

$$\Delta=(Z'_{\xi\xi}\bar{K}_p-\gamma^2\bar{m})(Z'_{\eta\eta}-\gamma^2\bar{J})-Z'_{\xi\eta}Z'_{\eta\xi} \tag{55}$$

The equation may be contracted as:

$$\begin{Bmatrix}\Delta\bar{F}_x\\ \Delta\bar{F}_y\end{Bmatrix}=\begin{Bmatrix}Z_{\xi\xi} & Z_{\xi\eta}\\ Z_{\eta\xi} & Z_{\eta\eta}\end{Bmatrix}\begin{Bmatrix}\Delta\bar{\xi}\\ \Delta\bar{\eta}\end{Bmatrix} \tag{56}$$

where the four impedances are obtained by multiplying out the matrices in equation (54).

Usually, the inertia of the pad can be ignored. With $\bar{m}=\bar{J}=0$, the effective impedances become:

$$Z_{\xi\xi}=\bar{K}_p(Z'_{\xi\xi}Z'_{\eta\eta}-Z'_{\xi\eta}Z'_{\eta\xi})/[(Z'_{\xi\xi}+\bar{K}_p)Z'_{\eta\eta}-Z'_{\xi\eta}Z'_{\eta\xi}]$$
$$Z_{\eta\eta}=Z_{\xi\eta}=Z_{\eta\xi}=0 \tag{57}$$

In this case, the pad has only radial stiffness and damping. Its radial impedance, $Z_{\xi\xi}$, is obtained as the oil film impedance: $Z'_{\xi\xi}-Z'_{\xi\eta}Z'_{\eta\xi}/Z'_{\eta\eta}$ in series with the pivot stiffness $\bar{K}_p$.

The impedance may be expressed as:

$$Z_{\xi\xi}=\bar{K}_{\xi\xi}+i\gamma\bar{B}_{\xi\xi} \tag{58}$$

where $\bar{K}_{\xi\xi}$ and $\bar{B}_{\xi\xi}$ are the effective stiffness and damping coefficients, respectively. They are not constants but depend strongly on frequency, even when the pad inertia is ignored.

To assemble the pads for the composite bearing, an X-Y coordinate system is introduced with origin in the bearing center and with the X-axis in the static load direction. The angle between the X-axis and the pad's ξ-axis is ψ, measured in the direction of rotation, whereby:

$$\begin{Bmatrix}\xi\\ \eta\end{Bmatrix}=\begin{Bmatrix}\cos\psi & \sin\psi\\ -\sin\psi & \cos\psi\end{Bmatrix}\begin{Bmatrix}X\\ Y\end{Bmatrix} \tag{59}$$

The reaction forces on the journal are:

$$\begin{Bmatrix}F_X\\ F_Y\end{Bmatrix}=\sum_{\text{pads}}\begin{Bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{Bmatrix}\begin{Bmatrix}F_x\\ F_y\end{Bmatrix} \tag{60}$$

The static equilibrium forces are F_{X0} and F_{Y0}, the former being equal to the static load, W, and the latter being zero. Because $\bar{F}_{y0}$ is zero, equation (60) yields:

$$\bar{F}_{X0}=W/\mu NDL\left(\frac{R}{C}\right)^2=\sum_{\text{pads}}\cos\psi\bullet\bar{F}_{x0} \tag{61}$$

$$\bar{F}_{Y0}=0=\sum_{\text{pads}}\sin\psi\bullet\bar{F}_{x0} \tag{62}$$

where $\bar{F}_{x0}$ is obtained from equation (20). $\bar{F}_{X0}$ gives the inverse Sommerfeld number for the bearing.

The static equilibrium coordinates for the journal center, X_0 and Y_0, must be selected such that equation (62) is satisfied. When the pads are arranged symmetrically with respect to the load line, Y_0 is zero, but otherwise some iterative scheme must be employed.

The dynamic forces may be expressed as:

$$\begin{Bmatrix}\Delta\bar{F}_X\\ \Delta\bar{F}_Y\end{Bmatrix}=\begin{Bmatrix}Z_{XX} & Z_{XY}\\ Z_{YX} & Z_{YY}\end{Bmatrix}\begin{Bmatrix}\Delta\bar{X}\\ \Delta\bar{Y}\end{Bmatrix} \tag{63}$$

where $\Delta\bar{X}$, $\Delta\bar{Y}$ = $\Delta X/C$, $\Delta Y/C$ and the forces are made dimensionless as also used in equation (19). By combining equations (56), (59), and (60) it is found that:

$$\begin{Bmatrix}Z_{XX} & Z_{XY}\\ Z_{YX} & Z_{YY}\end{Bmatrix}=\sum_{\text{pads}}\begin{Bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{Bmatrix}\begin{Bmatrix}Z_{\xi\xi} & Z_{\xi\eta}\\ Z_{\eta\xi} & Z_{\eta\eta}\end{Bmatrix}\begin{Bmatrix}\cos\psi & \sin\psi\\ -\sin\psi & \cos\psi\end{Bmatrix} \tag{64}$$

When the inertia of the pads is ignored such that equation (57) applies, the equation reduces to:

$$Z_{XX}=\sum_{\text{pads}}\cos^2\psi\bullet Z_{\xi\xi}\quad Z_{YY}=\sum_{\text{pads}}\sin^2\psi\bullet Z_{\xi\xi}$$

$$Z_{XY}=Z_{YX}=\sum_{\text{pads}}\cos\psi\bullet\sin\psi\bullet Z_{\xi\xi} \tag{65}$$

For a symmetric bearing, $Z_{XY}=Z_{YX}=0$.

To sum up the procedure, calculations are first performed for the single pad for a range of values of $\Gamma\bar{x}_0=\epsilon_0\cos\phi_0$. For each value the static equilibrium is determined such that $\bar{F}_{y0}$ is zero (equation (20)) after which equation (30) is solved to obtain $\Delta\bar{C}_0$ and, hence, Γ. Then F_{x0} can be computed from equation (20), and the radial motion of the pad is given by:

$$\bar{\xi}_{p0}=\xi_{p0}/C=F_{x0}/CK_{p0} \tag{66}$$

where K_{p0} is the static stiffness of the pivot. In case of Hertzian contact, the relationship is nonlinear and must be solved by some iterative procedure. The dynamic stiffness, K_p, is given by the slope of the load-deflection curve at static

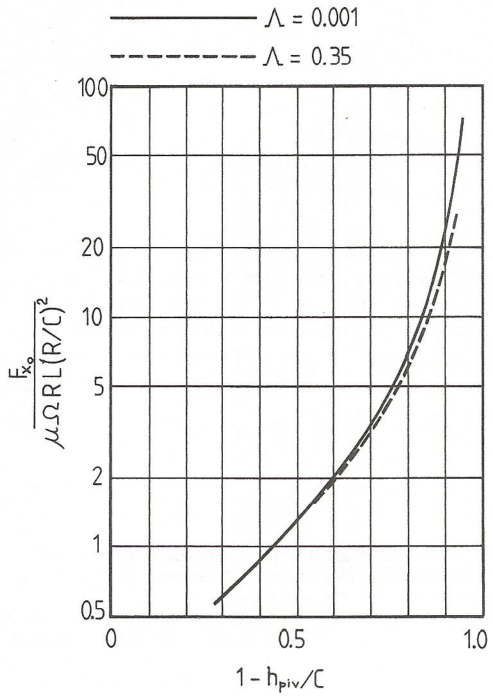

Fig. 2 Dimensionless pad load. 60 deg arc, *L/D* = 1, pivot position 0.6

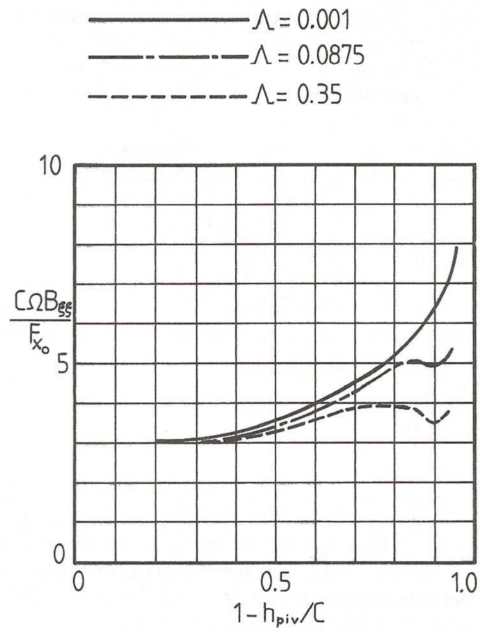

Fig. 4 Dimensionless damping coefficient. 60 deg arc, *L/D* = 1, pivot position 0.6, synchronous whirl frequency

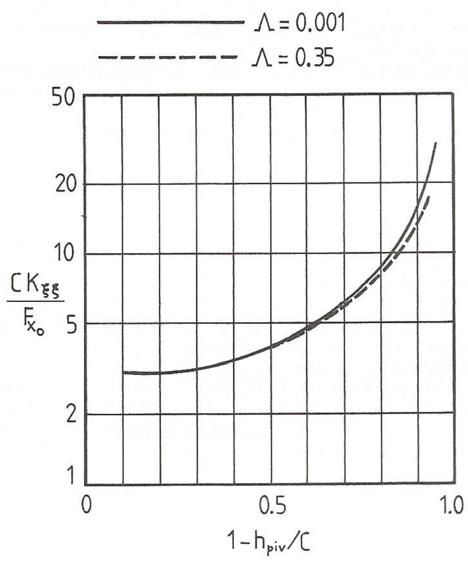

Fig. 3 Dimensionless stiffness coefficient. 60 deg arc, *L/D* = 1, pivot position 0.6, synchronous frequency

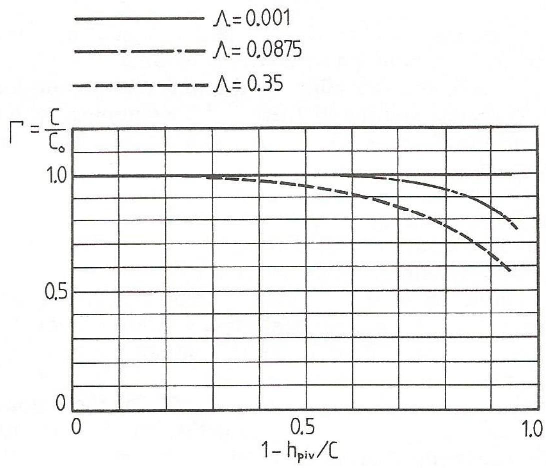

Fig. 5 Change in pad clearance caused by static load. 60 deg arc, *L/D* = 1, pivot position 0.6

equilibrium, and the impedances of the pad can then be obtained from equations (54) to (56), based on a specified frequency value.

Finally, equation (41) can be evaluated as:

$$\bar{\xi}_0 = \bar{x}_0 - \bar{d}_0 - \Delta\bar{C}_0 + \bar{\xi}_{p0} \tag{67}$$

Thus a table is generated where the load, F_{x0}, and the four impedances $Z_{\xi\xi}$, $Z_{\xi\eta}$, $Z_{\eta\xi}$, and $Z_{\eta\eta}$ are listed as functions of $\bar{\xi}_0$. By means of this table the pads can be assembled to the composite bearing.

For some chosen journal center position, X_0 and Y_0, the corresponding value of $\bar{\xi}_0$ is found from equation (59), and by interpolation in the table the pad load and impedances are obtained. They are summed in accordance with equations (61), (62) and (64), where equation (62) serves to check that the chosen position represents static equilibrium.

In the special case of a damped eigenvalue calculation or a rotor stability calculation, the frequency term, $i\omega$, must be replaced by the complex eigenvalue:

$$s = \lambda + i\omega$$

where λ is the damping exponent. In dimensionless form:

$$\frac{s}{\Omega} = -\frac{\delta}{2\pi} \cdot \gamma + i\gamma \tag{68}$$

where δ is the logarithmic decrement.

Discussion and Results

Because of the many parameters it would be neither useful nor feasible to present results in a form that is applicable in general. Even when the pad has uniform cross section it still requires 19 coefficients per $\epsilon_0 \cos\phi_0$ value to define the film properties (equations (21), (26), and (27)). Hence, in practice each case must be calculated separately, wherefore it is important to have a method available which is as economic as possible.

The present method accomplishes this by assuming that the influence from the pad deformation can be accounted for solely by the change in clearance, ignoring the details of the local deformations, and thereby eliminating the time-consuming true elasto-hydrodynamic solution.

The validity of the approximation has been tested for a 90 degree arc pad, $L/D = 1$ and pivot position: $(\pi-\theta_1)/(\theta_2-\theta_1) = 0.6$, under steady load with $\epsilon_0 = 0.95$ and $\Lambda = 0.35$. Even under such heavy loading conditions, the calculated load deviates only four percent from the value obtained by including the local deformations.

Further results are shown in Figs. 2 to 7. They apply to a 60 degree arc pad with $L/D = 1$, pivot position: $(\pi-\theta_1)/(\theta_2-\theta_1) = 0.6$, and frequency ratio $\gamma = 1$. As abscissa is chosen the dimensionless parameter:

$$1-\frac{h_{piv}}{C} = (C_0\epsilon_0\cos\phi_0 - \Delta C_0)/C \qquad (69)$$

where h_{piv} is the film thickness at the pivot point.

Figures 2, 3, and 4 can be compared with diagrams 1, 2, and 3 in [1] which are based on an elasto-hydrodynamic solution. The stiffness parameter $\bar{D}_e$ in reference [1] is the inverse of the flexibility parameter Λ such that $\bar{D}_e = 40$ for $\Lambda = 0.0875$, and $\bar{D}_e = 10$ for $\Lambda = 0.35$. The agreement is very good.

Figure 2 shows the static load on the pad, F_{x0}, in the dimensionless form $F_{x0}/\mu\Omega RL\ (R/C)^2$, equal to $\bar{F}_{x0}/\pi$ (equation (20)). Figures 3 and 4 give the dynamic radial stiffness and damping coefficients $K_{\xi\xi}$ and $B_{\xi\xi}$, respectively, in the dimensionless forms $CK_{\xi\xi}/F_{x0} = \bar{K}_{\xi\xi}/\bar{F}_{x0}$ and $C\,\Omega B_{\xi\xi}/F_{x0} = \bar{B}_{\xi\xi}/\bar{F}_{x0}$ (see equation (58)). The pad is assumed to have uniform cross section and its inertia is ignored.

As expected, the flexibility of the pad reduces the load-carrying capacity and the stiffness and the damping, with the reduction in damping the most pronounced. The corresponding change in clearance is shown in Fig. 5.

It should be emphasized that a similar reduction, of at least the same magnitude, is caused by the flexibility of the pivot support. Hence, both contributions must be included to obtain realistic results.

By treating the pad as a curved beam, as also done in references [1, 2], the analysis neglects any axial variation in the deformations. This is believed to be of minor importance for most pad geometries used in practice.

In the analysis, the radial clearance, C, for the unloaded pad includes any contribution from thermal deformations. Because the friction loss and, therefore, the pad temperature change with speed it would be natural to incorporate this contribution into the calculations. For a linear temperature gradient across the pad thickness with an average value of $\Delta T/H$, the change in radial clearance is:

$$\Delta C_{\text{thermal}} \simeq \alpha \cdot R_m^2 \cdot \left(\frac{\Delta T}{H}\right) \qquad (70)$$

where α is the coefficient of thermal expansion. In the absence, however, of a well-established relationship between friction loss and pad temperature, the thermal contribution is taken as implicit, rather than explicit, in the analysis.

Conclusion

An approximate method has been developed to calculate the stiffness and damping properties of a tilting pad journal bearing, including the influence of pad and pivot flexibility. The method is easy to program and is fast in execution. It can be used directly as a subroutine in rotor dynamic programs for calculating unbalance response or damped eigenvalues (stability).

References

1 Nilsson, L. R. K., "The Influence of Bearing Flexibility on the Dynamic Performance of Radial Oil Film Bearings," *Proceedings, The 5th Leeds-Lyon Symposium on Tribology*, 1978, pp. 311–319.

2 Hashimoto, H., Wada, S., and Marukawa, T., "Performance Characteristics of Large Scale Tilting-pad Journal Bearings," *Bull. J.S.M.E.*, Vol. 28, No. 242, 1985, pp. 1761–1767.

3 Klumpp, R., "Ein Beitrag zur Theorie von Kippsegmentlagern," Dissertation, University of Karlsruhe, 1975.

4 Malcher, L., "Die Federungs- und Dämpfungseigenschaften von Gleitlagern für Turbomaschinen," Dissertation, University of Karlsruhe, 1975.

5 Varga, Z., "Kippsegment-Radiallager 900 mm für Dampfturbogruppen: Eigenschaften und Erprobung," Brown Boveri Mitt., 6-77, 1977, pp. 309–320.

6 Knöss, K., "Gleitlager von Industrieturbogruppen," Brown Boveri Mitt., 5-80, 1980, pp. 300–308.

7 Qiande, M., Han, D.-C., and Glienicke, J., "Stabilitätseigenschaften von Gleitlagern bei Berücksichtigung der Lagerschalenelastizität," *Konstruktion*, Vol. 35, No. 2, 1983, pp. 45–52.

8 Lund, J. W., "Spring and Damping Coefficients for the Tilting Pad Journal Bearing," *Trans. ASLE*, Vol. 7, 1964, pp. 342–352.

9 Rouch, K. E., "Dynamics of Pivoted-Pad Journal Bearings, Including Pad Translation and Rotation Effects," *Trans. ASLE*, Vol. 26, 1983, pp. 102–109.

10 Parsell, J. K., Allaire, P. E., and Barrett, L. E., "Frequency Effects in Tilting-Pad Journal Bearing Dynamic Coefficients," *Trans. ASLE*, Vol. 26, 1983, pp. 222–227.

11 Nicholas, J. C., Gunter, E. J., and Barrett, L. E., "The Influence of Tilting Pad Bearing Characteristics on the Stability of High Speed Rotor-Bearing Systems," Topics in *Fluid Film Bearing and Rotor Bearing System Design and Optimization*, ASME, 1978, pp. 55–78.

12 Lund, J. W., and Thomsen, K. K., "A Calculation Method and Data for the Dynamic Coefficients of Oil-lubricated Journal Bearings," ibid., pp. 1–18.

13 Ericsson, U., "Temperature Distribution in the Oil Film of a Vibrating Tilting-Pad Bearing," Dissertation, Chalmers University of Technology, Gothenburg, Sweden, 1980.

14 Nicholas, J. C., and Barrett, L. E., "The Effect of Bearing Support Flexibility on Critical Speed Prediction," *Trans. ASLE*, Preprint No. 85-AM-2E-1, May 1985.

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